

Pseudo Magnetic Faraday and Quantum Hall Effect In Oscillating Graphene

Anita Bhagat* and Kieran Mullen†

Homer L. Dodge Dept. of Physics and Astronomy
The University of Oklahoma

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When a graphene layer is stressed, the strain alters the phase an electron accumulates hopping between sites in a way that can be modeled as arising from a pseudo-magnetic vector potential. We examine the case of an oscillating graphene ribbon and explore two new effects. The first is an oscillating resistance arising from an oscillating quantum Hall effect. This pseudo-magneto-resistance is large, and depends upon the frequency and the amplitude of the acoustic oscillations. Second, the acoustic oscillations can produce pseudo-magnetic fields on the scale of many Tesla that oscillate at high frequency, a regime inaccessible to real magnetic fields. One consequence should be a substantial pseudo-Faraday effect driving electrons in different valleys in opposite directions. In both cases, we make explicit calculations for experiment.

I. INTRODUCTION

In 2004 Geim *et al.* isolated a single layer of graphite, setting off the discovery of a long list of remarkable properties of graphene, such as its great strength, high mobility, and linear electronic energy spectrum.[1, 2] One additional extraordinary property it displays is a strain-induced pseudo-vector potential.[3] The deformation of the monolayer graphene lattice introduces strain which alters the phase difference between adjacent sites in a tight-binding model of the system. This phase difference can be viewed as arising from a “pseudo-magnetic field.”[9] Such a pseudo-magnetic field does not break time reversal symmetry because it couples with opposite sign to electrons occupying different Dirac points in the band structure. One consequence of this was dramatically demonstrated by the observation of quantized Landau levels in strained graphene in the absence of an external magnetic field.[6] This effect can be substantial: strain-induced vector potentials in static monolayer graphene have produced pseudo magnetic fields of 300 T.[7, 8]

There has been much theoretical and experimental work done to study the pseudo-magnetic field created by strain applied to a static graphene layer.[8] In this paper we will discuss the physics of a rapidly oscillating pseudo-magnetic field generated by an acoustically driven graphene ribbon. If models developed for static lattice distortions are valid for the relatively “slow” acoustic oscillations, the system can potentially have pseudo-magnetic fields of several Tesla oscillating at a kilohertz. Such a regime is inaccessible for normal magnetic fields and opens up many new possibilities.

In this work we start with a review of the connection between pseudo-magnetic fields and applied strain. We then investigate the theory of two physical consequences - the presence of an oscillating pseudo-quantum Hall effect

phenomena and valley charge polarization in low density graphene nano-ribbons. We conclude with a discussion of consequences for experiment.

A. Basic Theory

We consider an ideal 2D graphene nano-ribbon suspended between supports. The ribbon is driven acoustically by a piezoelectric transducer to produce a transverse standing wave. An atom at the position $\vec{r} \equiv x\hat{i} + y\hat{j} + z\hat{k}$ is displaced by vector $\vec{u}(\vec{r})$ (Fig.1)

$$\vec{u}(\vec{r}) = u_0 \sin(k_y y) \cos(\omega t) \hat{j} \quad (1)$$

producing a standing wave with displacements in the y direction. The stretching in the graphene layer produced by displacement produces a strain tensor $u_{ij}(\vec{r})$ which is given by

$$u_{xx} = \frac{\partial u_x}{\partial x}; \quad u_{yy} = \frac{\partial u_y}{\partial y}; \quad u_{xy} = \frac{1}{2} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \quad (2)$$

This strain tensor alters the phase difference between atoms in a tight-binding model of graphene in a fashion

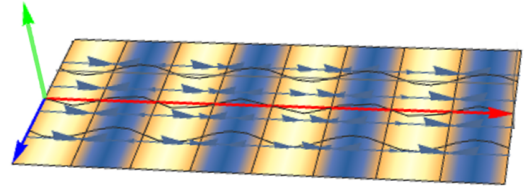


FIG. 1. A schematic representation of displacement in a suspended graphene nano-ribbon driven acoustically to create an oscillating pseudo-magnetic field, with the blue, red and green arrows representing the x , y and z directions, respectively. The displacement is in the y -direction, producing a vector potential in x and a magnetic field in the z direction. The amplitude of the magnetic field in the z -direction is denoted by the shading, with large positive fields shaded in yellow.

* anita@ou.edu

† kieran@ou.edu

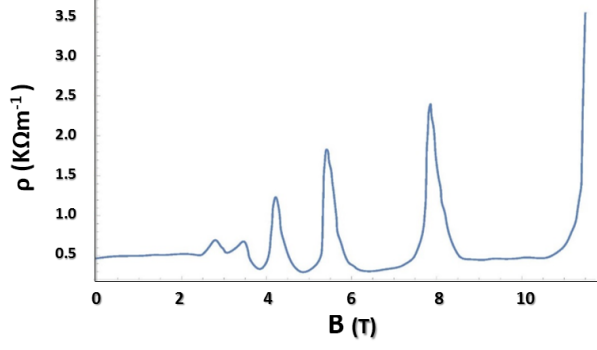


FIG. 2. Quantum Hall effect in 2D graphene nano-ribbon obtained by Janssen[10]. The longitudinal magneto-resistivity in 2D graphene is plotted as a function of real magnetic field.”Reproduced from [Janssen, T. J. B. M., et al. ”Quantum resistance metrology using graphene.” Reports on Progress in Physics 76.10 (2013): 104501.], with the permission of AIP Publishing.”

equivalent to one induced by a gauge field \vec{A}_s (pseudo-magnetic vector potential).[4, 5]

$$\vec{A}_s = \frac{c \hbar \beta \tau}{e a_0} \begin{bmatrix} u_{xx} - u_{yy} \\ -2u_{xy} \end{bmatrix} \quad (3)$$

where a_0 is the lattice constant and β and c are dimensionless parameters that are equal to 2 and 1 respectively.[9] The quantity τ is the pseudo-spin, taken as +1 at the K point in the 2D bandstructure, and -1 at the K' point. Therefore, even in the absence of a real magnetic field a strong pseudo-field is observed which comes from the curl of \vec{A}_s , but it is of opposite sign for electrons of opposite “valley-spin.” For the displacement of eq.(1) the magnetic field is given by

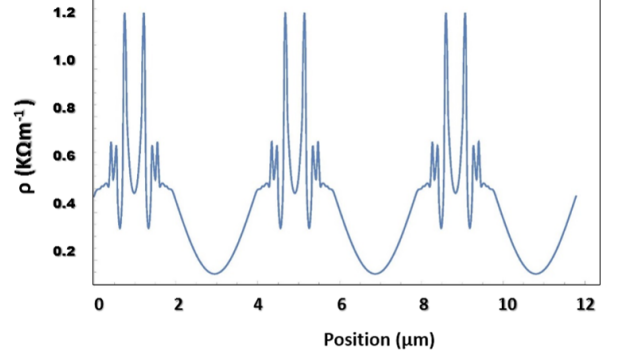
$$\vec{B}_s(\vec{r}) = \frac{-\hbar \beta \tau u_0 k_y^2}{e a_0} \sin(k_y y) \cos(\omega t) \hat{k} \quad (4)$$

We have introduced a dimensionless strain amplitude $f = \frac{\Delta L}{L}$. The strain amplitude is expressed as the ratio of the wave amplitude to a half wavelength, so that in terms of wave number, $f = \frac{u_0 k}{\pi}$.

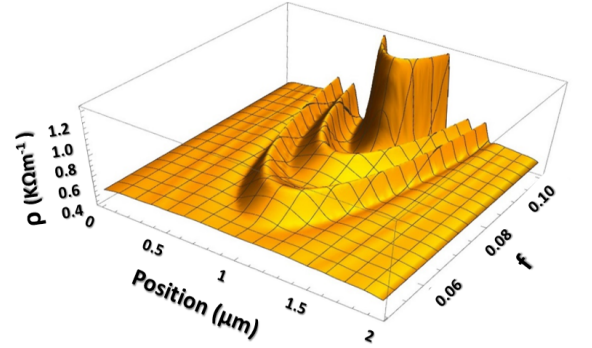
B. Oscillating quantum Hall magneto-resistance

We assume a suspended graphene nanoribbon set up for a two-terminal measurement, driven with an acoustic oscillation of a wavelength on the order of several microns. The oscillation has a wavelength much longer than the coherence length of electrons in the system, and a frequency far less than the characteristic electron-electron scattering rate. Therefore we treat each horizontal slice of the ribbon with displacement $\vec{u}(\vec{r})$ as an independent,

equilibrium, quantum Hall “sample” with a longitudinal resistance ρ_{xx} given by the value it would have in a *uniform* magnetic field $\vec{B}_s(\vec{u}(\vec{r}))$. Since the longitudinal resistance in quantum Hall systems is non-universal, we take a representative value from published experimental



(a)



(b)

FIG. 3. Longitudinal resistivity of an oscillating graphene nano-ribbon. (a) Resistivity as a function of position along the length of the monolayer graphene nano-ribbon for strain amplitude $u_0 = 0.1$, and wavelength $\lambda = 4 \mu\text{m}$ at a time $\omega t = 2\pi n$, for a sample assumed to show the longitudinal resistance of Fig.(2). (b) Plot of local resistivity as a function of strain and position for $\vec{B}_s(\vec{u}(\vec{r}))$ using interpolation of Fig.(2). The pseudo-magnetic field rises and then falls along the x-direction of the oscillating graphene nano-ribbon as seen in figure (a). The 3D plot shows this variation in the pseudo-magnetic field with the increase in strain amplitude f . When f is increased, more structures are visible, corresponding to the peaks at higher field in figure(a).

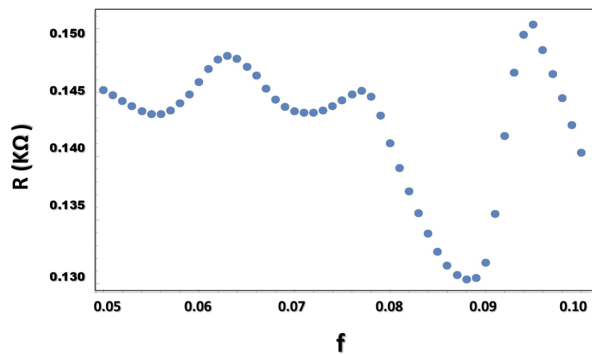


FIG. 4. (a) Plot of resistance amplitude at times when the oscillation is at a maximum, $\omega t = 2\pi n$, as a function of strain amplitude f . The total resistance is calculated by integrating the resistivity of Fig.(3) across one wavelength. The wavelength of the oscillation is $\lambda = 4.0\mu\text{m}$, and the resistance amplitude is calculated using the experimental data of ref.([10]).

data [10] on graphene in Fig.(2). The oscillating strain produces a pseudo-magnetic field standing wave with a maximum amplitude that potentially can reach several Tesla. The resistivity obtained from the interpolation of Fig.(2) will therefore also oscillate in space and time, (Fig.3). The oscillations of pseudo-field display more structure as the strain amplitude is increased, as shown in Fig.(3).

It is important to note that this effect does not break time reversal symmetry. Electrons from different K-points see opposite signs of the pseudo-magnetic field, but the magnetoresistance is independent of the sign of the field, so both pseudo-spins polarizations are gapped (or not gapped) at exactly the same value of the strain.

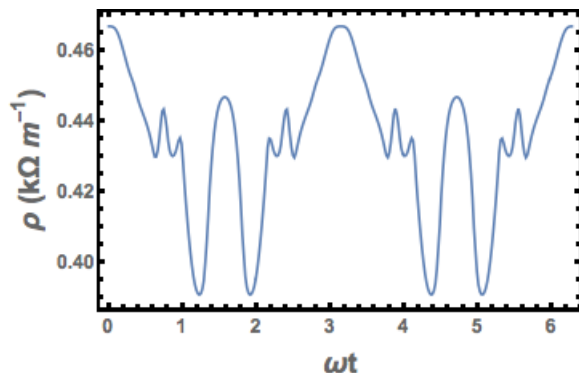


FIG. 5. Plot of the spatially averaged resistance of a nanoribbon as a function of time for a dimensionless strain amplitude, $f = \frac{u_0 k}{\pi} = 0.10$, where u_0 is the strain amplitude of a standing wave of wavenumber k . The resistance oscillates as a function of time because the effective strain amplitude, $f \cos(\omega t)$, oscillates in time. The resistance at any instant in time corresponds to integrating over a position slice of the resistivity plotted in Fig.(3b) for some value of this effective strain amplitude.

A more experimentally accessible property for a mechanically oscillating ribbon is its total resistance. For the oscillating monolayer graphene the total resistance at any point in time will be given by numerically integrating the resistivity graph of Fig.(3) along the length of the ribbon. If we choose a specific point in the oscillation, say when $\omega t = 2\pi n$, we can calculate the expected resistance at that point as a function of strain amplitude. This calculated resistance varies non-monotonically as a function of strain amplitude as shown in Fig.(4). It oscillates because increasing the strain amplitude may allow the pseudo-magneto-resistance to reach a higher resistance peak in Fig.(2), increasing the total resistivity, but increasing the strain further brings it into a regime of lower resistivity while simultaneously reducing the width of the higher resistance region. The width is reduced because the entire range of the sweep of the pseudo-magnetic field must still fit within one wavelength, and increasing the strain amplitude does not change the wavelength of the oscillation. The lower minima in the oscillations at larger strain are a reflection of the fact that the experimental data for ρ_{xx} has deeper minima at larger fields.

Alternatively, we may plot the resistance of the ribbon as a function of time as the standing wave goes through one oscillation. If Fig.(3a) represents a slice of Fig.(3b) at constant strain amplitude f , then the sinusoidal time dependence of the oscillation corresponds in effect to sweeping this slice from $f = 0$ to the some maximum value and back. To get the resistance at any time t we simply must integrate the corresponding slice along the x direction. An example of a resistance trace as a function of time for a given strain amplitude is given in Fig.(5)

Finally, we may plot the time average of the resistance of the ribbon as a function of strain amplitude, as shown in Fig.(6). As can be seen from above, much detail of the structure will be lost by integrating the resistivity over both space and time. However, this is the simplest and most straightforward measurement that would show this fundamental quantum mechanical effect from an acoustic oscillation.

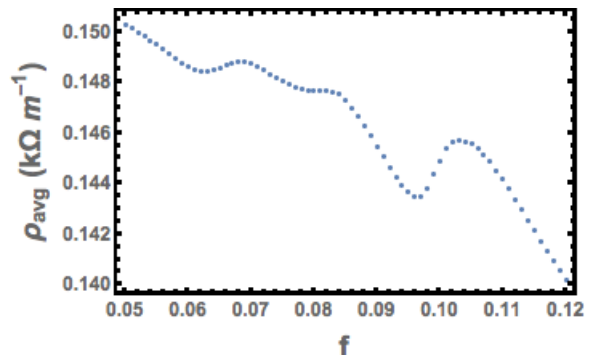


FIG. 6. Plot of time and spatially averaged resistance of a nanoribbon as a function of dimensionless strain amplitude, $f = \frac{u_0 k}{\pi}$ where u_0 is the strain amplitude of a standing wave of wavenumber k .

C. Pseudo-Faraday effect

As discussed above, a strain field can effectively act as a pseudo-magnetic field on a monolayer graphene. Since in our case the strain is oscillating in time, the pseudo-magnetic field is also oscillating in time. However any time dependence in the pseudo-magnetic field can be turned into a pseudo-electric field by a gauge transformation. In our case we have gauge transformation gives pseudo-electric field from pseudo-magnetic field as

$$\vec{E}_s(\vec{r}) = \frac{\hbar \beta \tau u_0 k_y \omega}{e a_o} \sin(k_y y) \sin(\omega t) \hat{i} \quad (5)$$

A pseudo-electric field (\vec{E}_s) of 5×10^4 volt m^{-1} can be observed for a $4 \mu m$ of wavelength and strain amplitude $f = 0.10$.

The pseudo field drives the electrons from K and K' valleys in opposite directions, producing no net current in the system. This must be true because time reversal symmetry has not been broken and a total net current is not generated. However, in a finite sample it might be possible for different pseudo-spin polarizations to “pile-up” on the boundary, with electrons at the K point driven to one edge, while those at K' driven to the other.

The result is that electrons of different valley polarization will accumulate on opposite edges of the ribbon leaving positive region on the interior. The ribbon will have a non-zero quadrupole moment. An order of magnitude estimate, assuming that all free charge is split into two piles at the edges of the system would give a voltage difference between the center and at the edges to be on the order of a hundred volts. However, this estimate neglects the $K \rightarrow K'$ scattering rates at the edges. If the rate is sufficiently high, the charge imbalance may be too small to measure.

We can remove the edge effect by driving the ribbon with a more structured wave. If the graphene nano-ribbon is driven acoustically with displacement $\vec{u}(x, y, z) = u_0 \cos(k_x x) \cos(k_y y) \cos(\omega t) \hat{j}$, the pseudo-electric field is

$$\vec{E}_s(\vec{r}) = \frac{\hbar \beta \tau u_0 \omega}{e a_o} \left[k_x \sin(k_x x) \cos(k_y y) \hat{i} - \frac{k_y}{2} \cos(k_x x) \sin(k_y y) \hat{j} \right] \sin(\omega t) \quad (6)$$

The pseudo-electric field has a non-zero divergence in opposite directions for opposite valleys electrons as seen in Fig.(7). Again, the field is in opposite directions for electrons at different K -points, but in this case it would lead to a square lattice of charge, with the K and K' electrons forming an interlaced checkerboard of charge. Again, the amount of charge imbalance would depend upon the $K \rightarrow K'$ scattering rate, but in this case there is no edge to act as a mechanism for scattering.

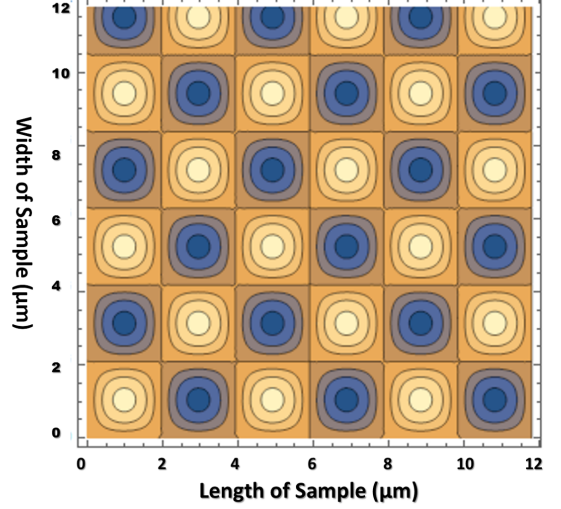


FIG. 7. The divergence pseudo-electric field as a function of nano-ribbon graphene length and width at $t = 0$. Light regions correspond to positive divergence and dark to a negative divergence. The diverging pseudo-electric field is observed in opposite directions for the electrons from K and K' valleys. At low charge densities, the large pseudo-electric field may drive the charge distribution to form puddles that appear and disappear as a function of time.

D. Conclusion

Experiments have already verified that a static strain in graphene can produce a pseudo-magnetic field of many Tesla. If the static distortion calculation is valid for the “slow” distortion of an acoustic wave, then pseudo-magnetic fields of several Tesla could be observed to oscillate at high frequency, a previously inaccessible regime of electron dynamics. We have investigated two resulting phenomena - an oscillation in the resistance due to a quantum Hall-like effect, and a Faraday-like effect produced from time dependent pseudo-magnetic field. These phenomena should be observable at experimentally accessible frequencies and temperatures. If these phenomena are not observed, then the cross-over from the experimentally validated static theory to a time dependent distortion must itself be investigated.

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